MECHANICS OF SOLIDS (ME F211)

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Chapter-3

Forces and Moments Transmitted by Slender Members

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- Determination of Forces and moments under point loads
- Sign conventions for shear force and Bending moment
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Forces and Moments Transmitted by Slender Members

Slender members

These are load carrying elements having much greater length (at least five times) than its lateral dimensions. Slender members can be pulled, bent and twisted

Examples: beam, columns, shafts, rods, stringers, struts, and links.

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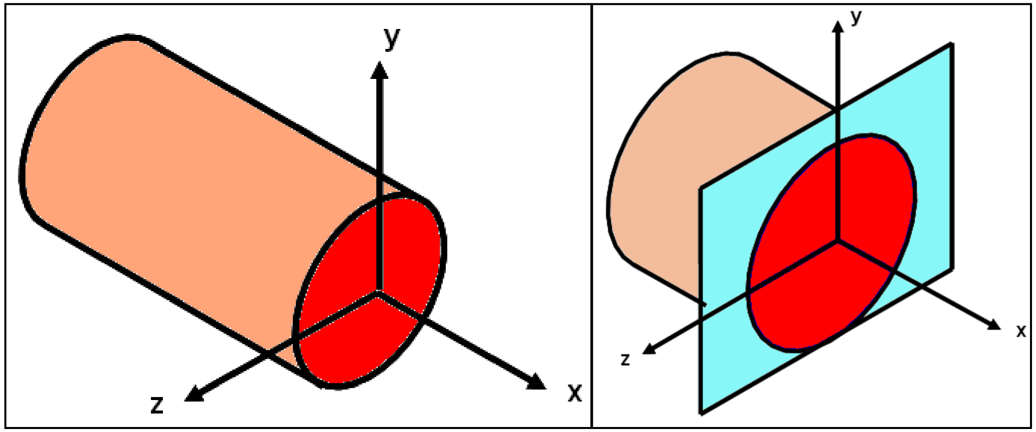
GENERAL METHOD TO FIND INTERNAL FORCES AND MOMENTS

- A general method for determining the internal forces and moments acting across any section of a slender member which is in equilibrium is to cut and that part & isolate from the system.
- The isolated part is in equilibrium. Apply the equations of equilibrium to find internal forces and moments.

- x- Section : The section normal to x- axis
- y- section : The section normal to y- axis
- z- section : The section normal to z- axis



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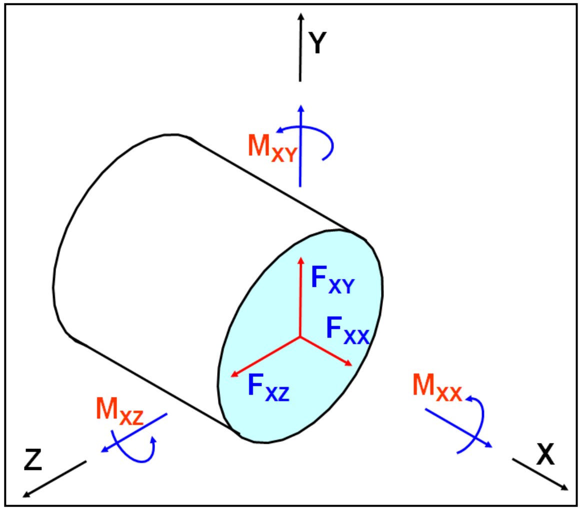
Forces and moments acting on a cross section of a member

- F_{XX}: Force acting on x-section and is along x- axis
- Similarly F_{XY}, F_{XZ}, M_{XX}, M_{XY} and M_{XZ} can be explained



<u>NOMENCLATURE</u>

- F_{XX} (Axial force) : This component tends to elongate the member and is often given the symbol *F* or F_x.
- F_{XY} & F_{XZ} (Shear force) : These components tend to shear one part of the member relative to the adjacent part and are often given the symbols V, or V_Y and V_Z

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- M_{XX} (Twisting moment) : This component is responsible for the twisting of the member about its axis and is often given the symbol M_T or $M_{TX.}$
- M_{XY} & M_{XZ} (Bending moments): These components cause the member to bend and are often given the symbols M_b, or M_{by} and $M_{bz.}$

Positive face of given section

If the outward normal points in a positive coordinate direction then that face is called as positive face

Negative face of given section

If the outward normal points in a negative coordinate direction then that face is called as Negative face

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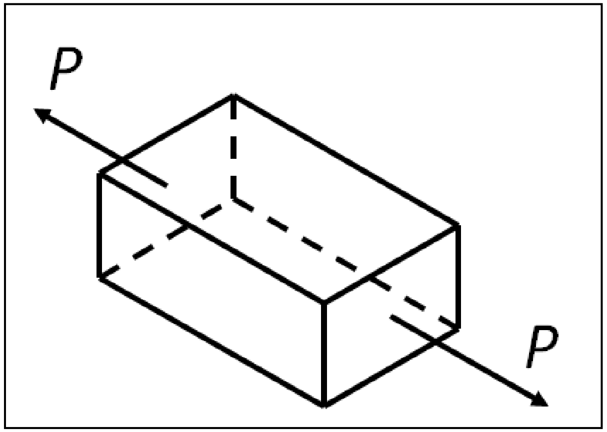
Sign convention for the axial force, shear force, and bending moment.

- If force or moment component acts on a positive face in a positive coordinate direction then these components are treated as *positive*
- If force or moment component acts on a negative face in a negative coordinate direction then these components are treated as *positive*

Positive Force (Tensile)



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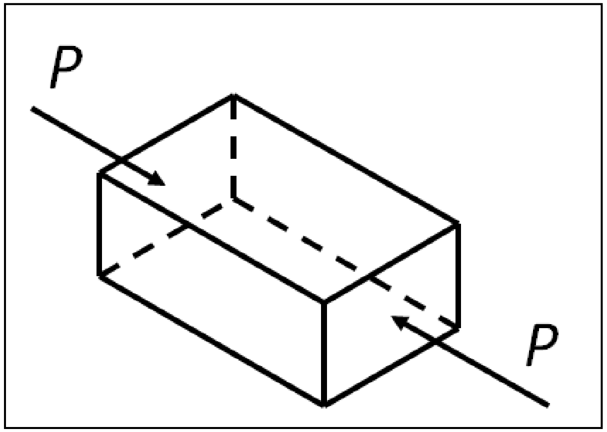
Sign convention for the axial force, shear force, and bending moment.

- If force or moment component acts on a positive face in a negative coordinate direction then these components are treated as *negative*
- If force or moment component acts on a negative face in a positive coordinate direction then these components are treated as *negative*

Negative Force (Compressive)



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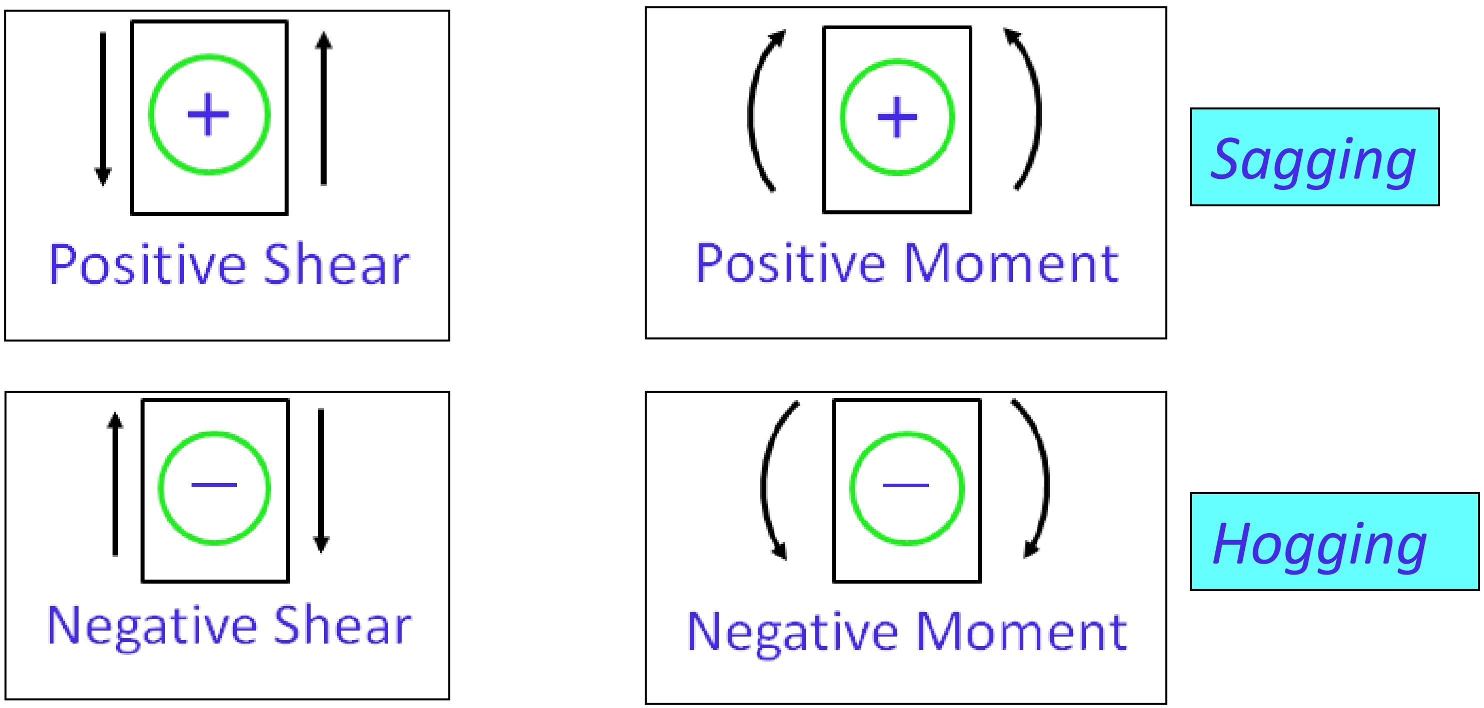
Sign convention for the axial force, shear force, and bending moment.

Shear force (V) Bending moment (M_b)



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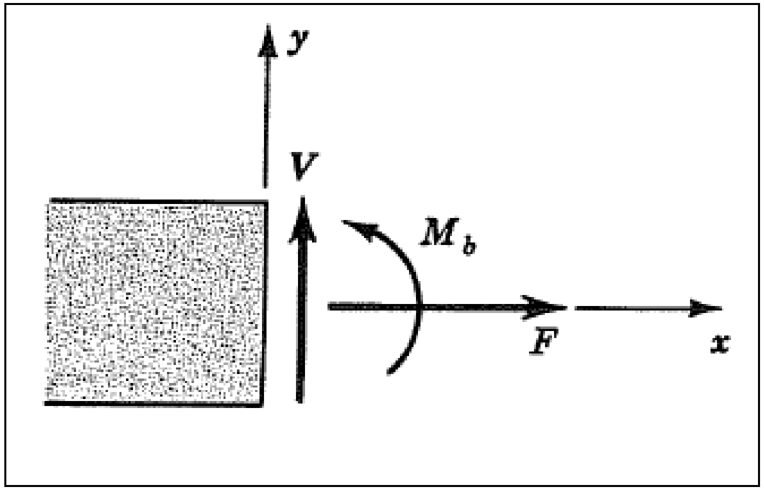
If the plane of loading is the *x-y* plane then only three components occur:

- The axial force F_{xx} (F), the shear force F_{xy}, (V), and the bending moment M_{xz} (M_b),



Force and moment components in two dimensions innovate achieve lead

The steps involved in solving the problems

1. Idealize the actual problem, i.e., create a model of the system, and isolate the main structure, showing all forces acting on the structure.
2. Using the equations of equilibrium calculate unknown external forces or support reactions

$$\Sigma F = 0 \quad \& \quad \Sigma M = 0$$

3. Cut the member at a point of interest, isolate one of the segments, and repeat step 2 on that segment.

Shear Force Diagram (SFD) and Bending Moment Diagram (BMD):

Graphs that show shear force and bending moment plotted against distance along beam are called as shear force and bending moment diagram respectively

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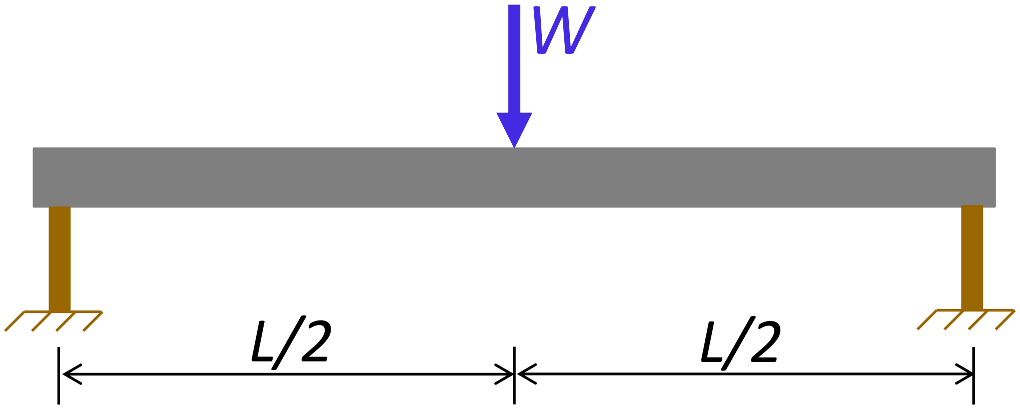
Problem:

For given loading diagram, determine the shear force and bending moment at all salient points. Plot the distribution of shear force and bending moment along the length of the beam.



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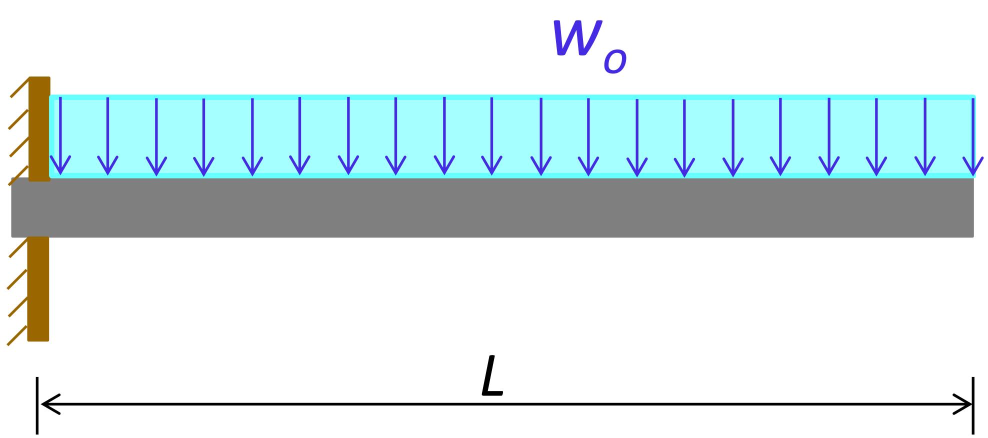

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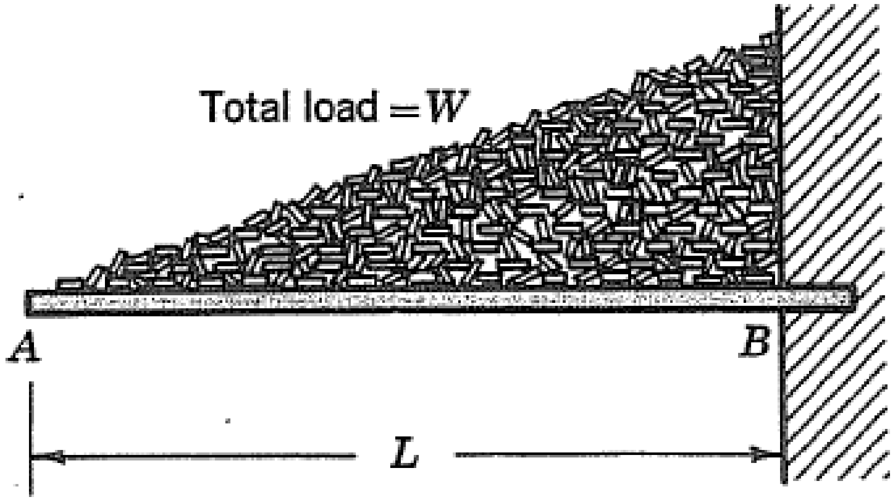

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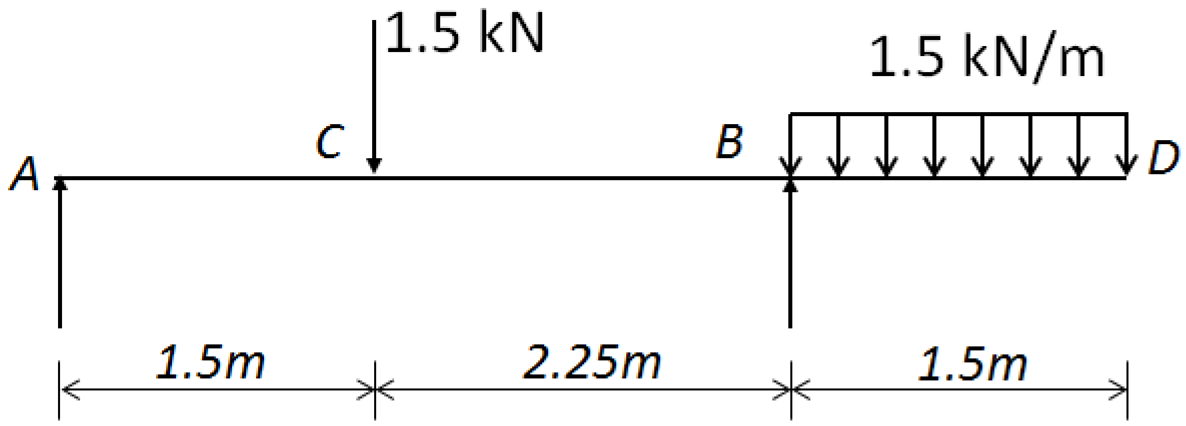
Problem:

For given loading diagram, determine the shear force and bending moment at all salient points. Plot the distribution of shear force and bending moment along the length of the beam. Also determine the position of point of contraflexure, if any.



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Differential Equilibrium relationships

1. It is an alternative procedure for obtaining internal forces and moments for the slender members
2. Instead of cutting a beam in two and applying the equilibrium conditions to one of the segments, a very small differential element of the beam will be considered
3. The conditions of equilibrium combined with a limiting conditions will lead us to differential equations connecting the load, the shear force, and the bending moment.
4. Integration of these relationships for particular cases furnishes us with an alternative method for evaluating shear forces and bending moments.

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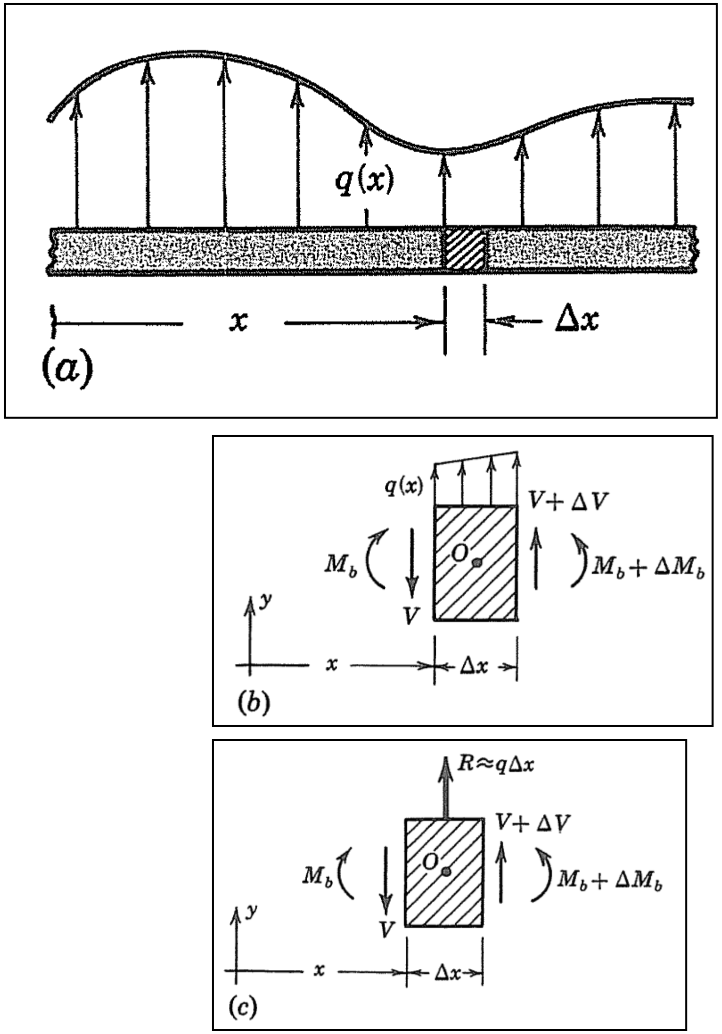
If the variation of *q(x)* is smooth and if Δx is very small then *R* is very nearly given by $q\,\Delta x$ and the line of action of *R* will very nearly pass through the midpoint '*O*' of the element.

Use of equilibrium equations to differential element will deduce following relations

$$\frac{dV}{dx} + q = 0 \quad \text{and} \quad \frac{dM_b}{dx} + V = 0$$

Integration of above equations with appropriate boundary conditions will give values of shear forces and bending moments



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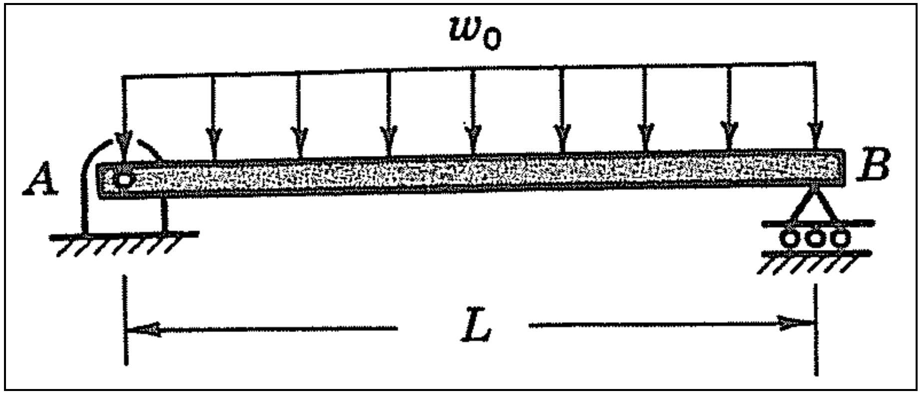
Problem

In Figure a beam is carrying a uniformly distributed load of intensity $q = -w_o$ is supported by a pinned joint at *A* and a roller support at *B.* We shall obtain shear-force and bending-moment diagrams by integration of the differential relationships.



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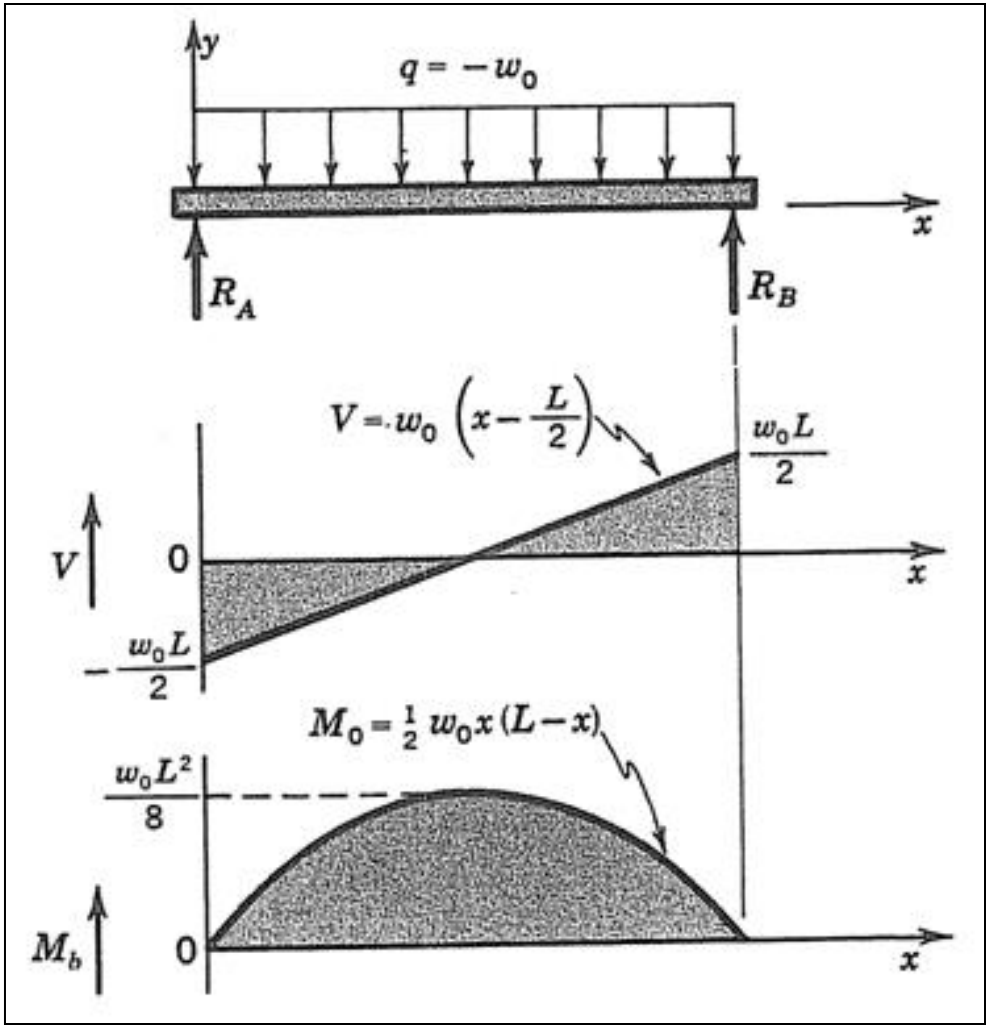
Solution



Loading Diagram

Share Force Diagram

Bending Moment Diagram

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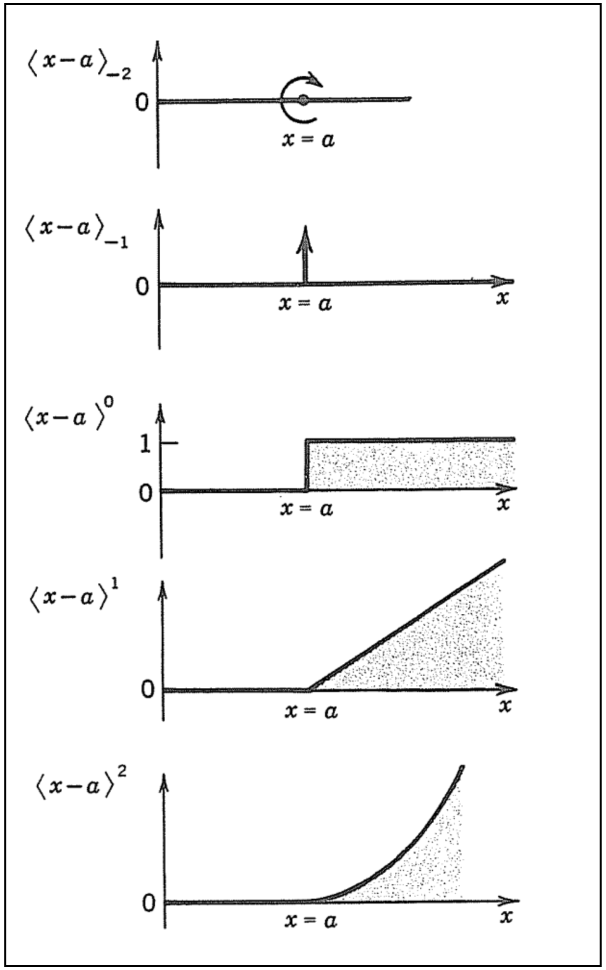
SINGULARITY FUNCTIONS

- This is special mathematical tool to handle discontinuous load functions
- Figure shows family of singularity functions specially designed for this purpose

$$f_n(x) = \langle x-a \rangle^n \begin{cases} = 0; \text{ if } x < a \\ = (x-a)^n; \text{ if } x > a \end{cases}$$

- The function $\langle x - a \rangle^0$ is called unit step starting at $x = a$.
- The function $\langle x - a \rangle^1$ is called unit ramp starting at $x = a$.



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- The first two members of the family shown in figure are exceptional. To emphasize this, the exponent is written below the bracket instead of above.
- The function $\langle x - a \rangle_{-1}$ is called unit concentrated load or unit impulse function acting at $x = a$.
- The function $\langle x - a \rangle_{-2}$ is called unit concentrated moment or unit doublet function acting at $x = a$.
- These functions are zero everywhere except at $x = a$ where they are infinite.
- They are, however, infinite in such a way that integration of unit concentrated moment is unit concentrated load and integration of unit concentrated load is unit step function.

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- Integration law for unit step and unit ramp functions is

$$\int_{-\infty}^{x} \langle x-a \rangle^{n}\, dx = \frac{\langle x-a \rangle^{n+1}}{n+1} \qquad \text{for} \quad n \geq 0$$

- Integration law for unit concentrated load and unit concentrated moment is

$$\int_{-\infty}^{x} \langle x-a \rangle^{n}\, dx = \langle x-a \rangle^{n+1} \qquad \text{for} \quad n < 0$$

Unit concentrated moment $$\int_{-\infty}^{x} \langle x-a \rangle_{-2}\, dx = \langle x-a \rangle_{-1}$$

Unit concentrated load $$\int_{-\infty}^{x} \langle x-a \rangle_{-1}\, dx = \langle x-a \rangle^{0}$$

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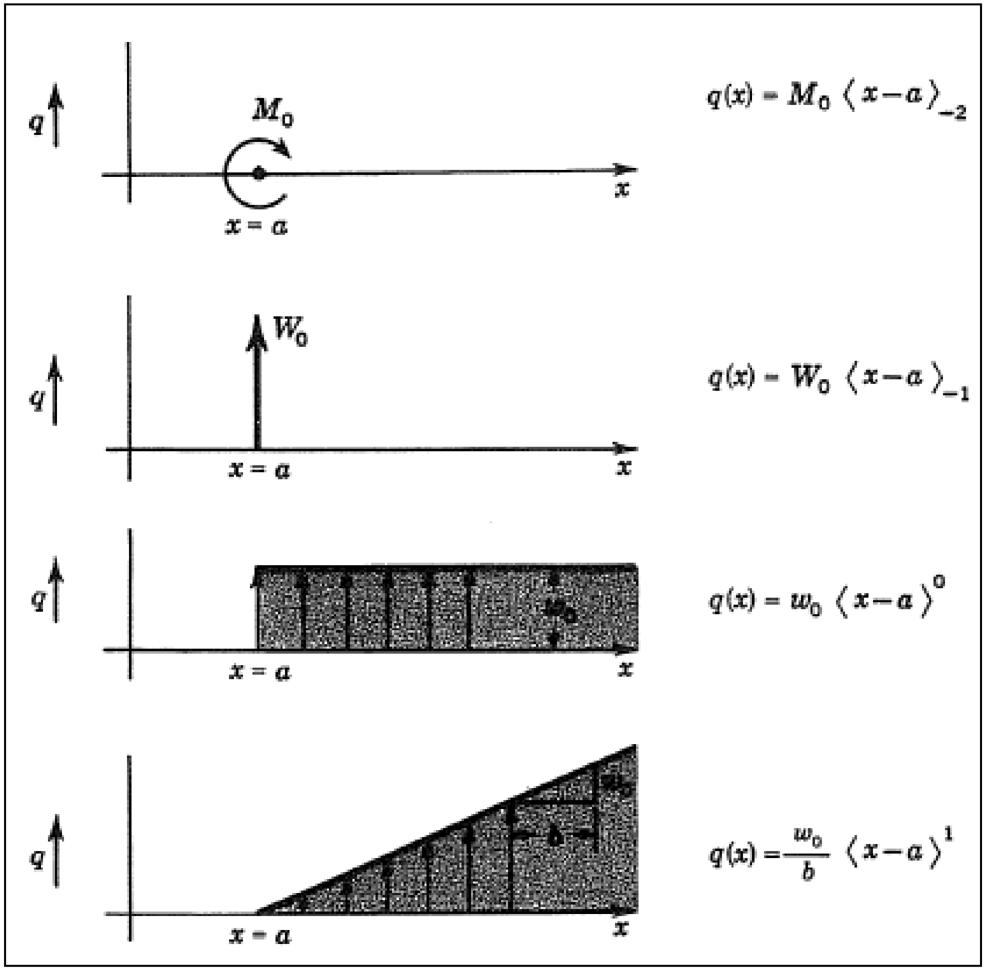
Loading intensities represented by Singularity functions



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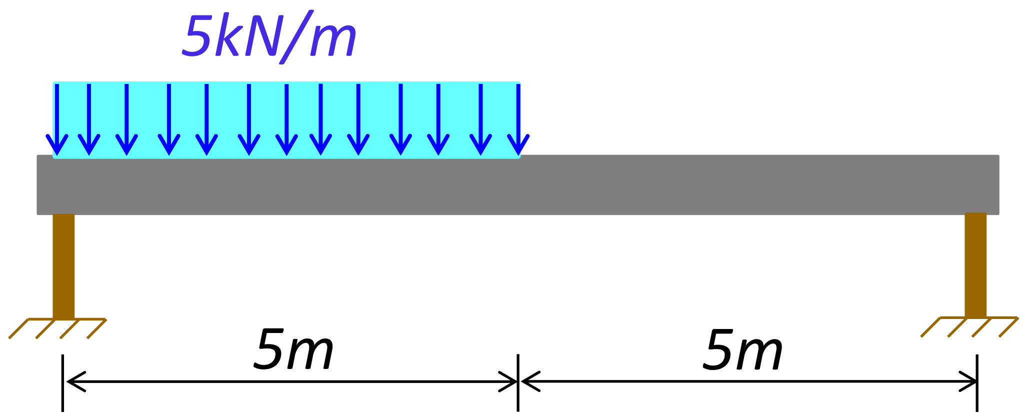
Problem:

For given loading diagram, determine the shear force and bending moment at all salient points. Plot the distribution of shear force and bending moment along the length of the beam.

Note: Use Singularity functions



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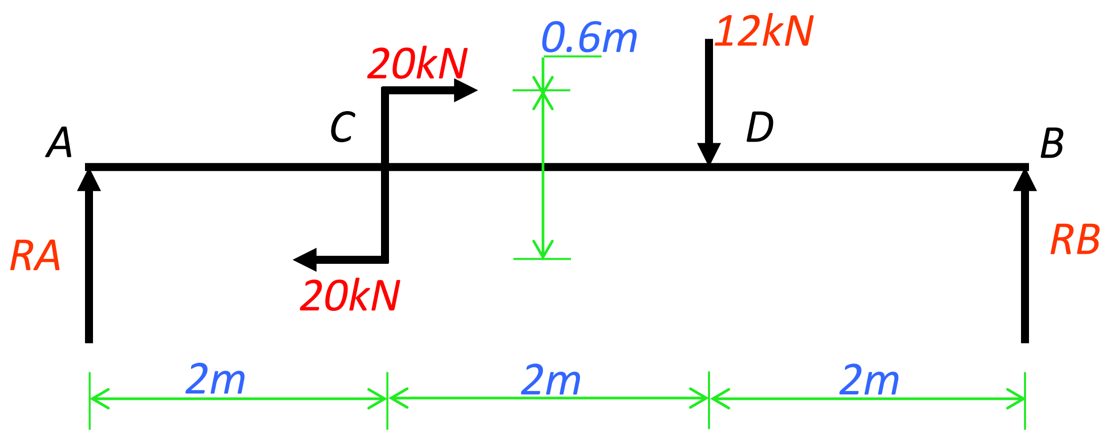

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References

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